Close Wed: HW_5A,5B,5C   (6.5,7.1,7.2)

Office Hours: 2:00-3:30 in COM B-006

$$2. \int_0^1 x^2 e^{x/3} dx$$

7.1 Integration by Parts *(continued)*

Summary: $\int u\, dv = uv - \int v\, du$

1. Pick u = ??. The rest is dv.
2. Compute du and v.

Entry Task: Evaluate

$$1. \int \frac{\ln(x)}{\sqrt{x}}\, dx$$

Integration by parts is good for:

Products: $xe^x, x^2 \cos(3x), x\sin(5x)$

Logs: $\ln(x), x^{10}\ln(x), \dfrac{\ln(x)}{x^3}, \ldots$

Inv. Tri: $\sin^{-1}(x), x\tan^{-1}(x), \ldots$

Products: $e^x\sin(x), e^x\cos(x)$

Example:

$$\int \tan^{-1}(x)dx = \int \arctan(x)dx$$

Example: (Never ending integration by parts and how to end it):

$$\int e^x \cos(x) \, dx$$

7.2 Trigonometric Integral Methods

Goal: A procedure to integrate *any* combination of trig functions.

Motivating examples: **These are substitution problems, what is u?**

$$\int \sin^3(x)\,(1 - \sin^2(x))\cos(x)\,dx$$

$$\int (1 - \cos^2(x))\cos^5(x)\sin(x)\,dx$$

$$\int \tan^5(x)\,(1 + \tan^2(x))\sec^2(x)\,dx$$

$$\int \sec^6(x)\sec(x)\tan(x)\,dx$$

7.2 idea: Use trig identities to turn almost all trig problems into one of these situations!

Essential Tools

$$\tan(x) = \frac{\sin(x)}{\cos(x)}, \cot(x) = \frac{\cos(x)}{\sin(x)},$$
$$\sec(x) = \frac{1}{\cos(x)}, \csc(x) = \frac{1}{\sin(x)}.$$

See my online postings (or the Appendix of your book) for a more general discussion and proofs of trig identities.

$$\sin^2(x) + \cos^2(x) = 1$$
$$\tan^2(x) + 1 = \sec^2(x)$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$
$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$
$$\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$$

Case 1 (*cos(x) or sin(x) has odd power*)

$i)$ $\displaystyle\int \sin^2(x)\cos^3(x)\,dx$

$ii)$ $\displaystyle\int \sin^3(x)\,dx$

Case 2 (*both sin(x), cos(x) even powers*)

$i)\ \displaystyle\int \cos^2(x)\,dx$

$ii)\ \displaystyle\int \sin^4(x)\,dx$

Case 3 (*even power on sec(x)*)

$$\int \tan^2(x) \sec^4(x)\, dx$$

Case 4 (*odd power on tan(x), and at least one sec(x)*)

$$\int \tan^3(x) \sec^5(x)\, dx$$

Notes: And if you've tried all methods
and are stuck, here are things to try:
1. Rewrite in terms of sin(x) and cos(x).
2. Rewrite in terms of sec(x) and tan(x).
3. Try using trig identities.

There are still a few "holes".
Particularly, odd power on sec(x).
For these you can quote
(proof in the book):

$$\int \tan(x) \, dx = \ln|\sec(x)| + C$$

$$\int \sec(x) \, dx = \ln|\sec(x) + \tan(x)| + C$$

$$\int \sec^3(x) \, dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln|\sec(x) + \tan(x)| + C$$

What is the first step in each?

$$\int \sin^3(x)\cos^4(x)dx$$

$$\int \sin^5(x)\cos^3(x)dx$$

$$\int \cos^4(x)dx$$

$$\int \tan^5(x)\sec^4(x)dx$$

$$\int \tan^5(x)\sec(x)dx$$